YITP-96-19

Are nontopological strings produced at the electroweak phase transition ?

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Abstract

We formulate a local condition for a nontopological defect to be present. We apply it for electroweak strings and estimate the probability of their existence at the Ginzburg temperature. As a result we find strings long enough to serve for baryon-number generation are unlikely to be produced.

98.80.Cq, 11.27.+d

Typeset using REVTEX

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Topological defects are produced at cosmological phase transitions if vacuum structure after the symmetry breaking is nontrivial [1,2]. Even when it is trivial, however, nontopological defects might be produced. One of the well-known examples is an electroweak string [3]. It has a string-like configuration of the false vacuum which satisfies field equations of the minimal standard electroweak model, although whether it constitutes a local energy minimum is still under investigation [4]. While topologically stable strings have also been proposed under the non-standard extension of the theory [5], we concentrate on the possibility of nontopological strings within the standard model here.

The electroweak strings might be useful for baryogenesis in our Universe [6,7]. They can generate an out-of-equilibrium state even if the electroweak phase transition is of the second order. Moreover the electroweak strings themselves have baryon number and may contribute to the baryon asymmetry production [8] or they can induce baryon-number fluctuations through interaction with background electromagnetic fields [9]. Their effect on the sphaleron transition rate has been discussed in [10].

All the above analyses, although interesting, rely on the assumption that the nontopological strings are indeed produced at the electroweak phase transition more or less in a similar manner to ordinary topological strings. However, a more careful analysis is required, since there is no topological reason for electroweak strings to extend without an end but they may have a finite length with a monopole-like configuration at one end and an antimonopole-like configuration at the other. Although much work has been done on the stability of the width of an infinitely long electroweak string [4], no one has really estimated their formation rate at the phase transition except for a preliminary treatment [11] in which the authors concerned mostly with the validity of the geodesic rule in the transient region between different phases. But their approach is inappropriate to apply for the present problem, since the number density of the electroweak strings cannot be calculated only by the phase distribution of the Higgs field since a nonvanishing winding number alone does not guarantee the existence of a false vacuum region and it must be imposed as an extra condition. Even if the infinitely long string solution is stable against perturbation on its width, we cannot say strings are

indeed produced at the electroweak epoch. Such stability may help their survival after formation, but their initial number density must be determined by the realization probability of string-like configuration at the phase transition. In the present *Letter* we estimate the formation probability of the electroweak strings, along which the Higgs fields have a vanishing amplitude, at the end of the phase transition.

First, for comparison, let us consider the case of an ordinary topological cosmic string which is produced when local U(1) symmetry breaks down. In this model, the Higgs field, Φ, is a complex scalar written by

$$\Phi = \Phi_1 + i\Phi_2 \ , \tag{1}$$

where Φ_1 and Φ_2 are real. As is well known, if the phase of Φ is randomly distributed on each correlated region, there should be 0.25 string per one correlation volume [12]. This method, however, cannot be applied to the case of the nontopological electroweak string since even if any winding number around a certain region exists, this does not necessarily imply that a false vacuum is trapped in it. Therefore we start with discussing the condition for a gauged U(1) string to be present without resorting to such topological consideration.

The cosmic string can be regarded as a line-like region where the amplitude of Φ equals zero. Thus the condition that a string exists at a certain point in the universe, $\vec{x} = \vec{a}$, is $\Phi_j(\vec{a}) = 0$ $(j = 1, 2)$ and at the same time there exists a neighboring point, $\vec{a} + \vec{\varepsilon}$, where $\Phi_j(\vec{a} + \vec{\varepsilon}) = 0$ $(j = 1, 2)$ hold, too. Since we can always set one of the components of the Higgs field equal to zero at $\vec{x} = \vec{a}$ using a gauge transformation, the first condition reduces to having the other component to be zero, too. On the other hand, $|\vec{\varepsilon}|$ is small by definition, so the second condition may be rewritten as

$$\Phi_j(\vec{a} + \vec{\varepsilon}) = \Phi_j(\vec{a}) + \vec{\varepsilon} \cdot \vec{\nabla}\Phi_j(\vec{a}) = \vec{\varepsilon} \cdot \vec{\nabla}\Phi_j(\vec{a}) = 0 \ (j = 1, 2) \ , \tag{2}$$

that is, there should exist a spatial vector $\vec{\varepsilon}$ orthogonal to both $\vec{\nabla}\Phi_1(\vec{a})$ and $\vec{\nabla}\Phi_2(\vec{a})$. But one can always find such a vector simply by choosing a normal vector to the plane defined by $\vec{\nabla}\Phi_1(\vec{a})$ and $\vec{\nabla}\Phi_2(\vec{a})$. Thus once we find $\Phi_j = 0$ at $\vec{x} = \vec{a}$, a line-like configuration of the

false vacuum extends without an end, which is a consequence of the topological structure of the vacuum manifold of the Abelian Higgs model.

Before proceeding to the case of the electroweak string, here we consider a rather inconceivable possibility of domain wall formation in the above model. A domain wall configuration can be easily shown to exist as a nontopological defect in this model, for example, by a distribution such that $\Phi_2(\vec{x}) = 0$ everywhere and that $\Phi_1(\vec{x})$ obeys a similar solution as a domain wall in a model of a real scalar field.

The condition that a domain wall exists at $\vec{x} = \vec{a}$ is, in addition to having $\Phi_j(\vec{a}) = 0$, there should exist two linearly independent spatial vectors, $\vec{\varepsilon}_1$ and $\vec{\varepsilon}_2$, which satisfy $\Phi_j(\vec{a} + \vec{\varepsilon}_n) = 0$ $(n = 1, 2)$ or

$$\vec{\varepsilon}_n \cdot \vec{\nabla}\Phi_j(\vec{a}) = 0 \; (n = 1, 2; \; j = 1, 2) \; . \tag{3}$$

The necessary and sufficient condition for it is that $\vec{\nabla}\Phi_1(\vec{a})$ and $\vec{\nabla}\Phi_2(\vec{a})$ are parallel to each other including the trivial case that one or the both of them have a vanishing amplitude. We can also show that the above condition is gauge-invariant. In fact, since gauge-transformation is a linear transformation for the Higgs fields such as

$$\Phi'_j(x) = \sum_l c_{jl}(x)\,\Phi_l(x) \;\; , \tag{4}$$

we find

$$\vec{\nabla}\Phi'_j(x) = \sum_l \vec{\nabla}c_{jl}(x) \cdot \Phi_l(x) + \sum_l c_{jl}(x)\,\vec{\nabla}\Phi_l(x) \;\; , \tag{5}$$

but at $\vec{x} = \vec{a}$ we have $\Phi_l(\vec{a}) = 0$ by assumption, so

$$\vec{\nabla}\Phi'_j(\vec{a}) = \sum_l c_{jl}(\vec{a})\,\vec{\nabla}\Phi_l(\vec{a}) \;\; , \tag{6}$$

which implies the gauge-invariance of (3). Furthermore we can choose the spatial coordinate at $\vec{x} = \vec{a}$ such that $\vec{\nabla}\Phi_1(\vec{a})$ has a nonvanishing component only in the x-component. Then the condition (3) reduces to

$$\partial_y\Phi_2(\vec{a}) = \partial_z\Phi_2(\vec{a}) = 0 \; . \tag{7}$$

Thus in this case two additional conditions must be satisfied to produce a nontopological defect.

Now we return to the electroweak string. In the minimal standard model, the Higgs field, ϕ, is an SU(2) doublet and we write it as

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} , \tag{8}$$

where ϕ_j $(j = 1, 2, 3, 4)$ is a real component. Similarly to the case of the Abelian Higgs model, the conditions for the existence of a string at $\vec{x} = \vec{a}$ are that $\phi_j(\vec{a}) = 0$ $(j = 1, 2, 3, 4)$ and that there exist an infinitesimal spatial vector $\vec{\varepsilon}$ such that $\phi_j(\vec{a} + \vec{\varepsilon}) = 0$ $(j = 1, 2, 3, 4)$. Since we can rotate ϕ using a gauge transformation so that only one component is nonvanishing at $\vec{x} = \vec{a}$, the first condition reduces to that the remaining component is also vanishing, whose probability is denoted by p_0 hereafter. The second condition reads

$$\vec{\varepsilon} \cdot \vec{\nabla}\phi_j(\vec{a}) = 0 \ (j = 1, 2, 3, 4) , \tag{9}$$

which can again be shown to be gauge-invariant.

For a nontrivial solution of $\vec{\varepsilon}$ to exist, it is necessary and sufficient that all the vectors $\vec{\nabla}\phi_j(\vec{a})$ lie in the same plain defined by two linearly independent vectors, say, $\vec{\nabla}\phi_1(\vec{a})$ and $\vec{\nabla}\phi_2(\vec{a})$. Then a normal vector to that plain can serve as $\vec{\varepsilon}$ and the remaining conditions turns out to be

$$\vec{\varepsilon} \cdot \vec{\nabla}\phi_3(\vec{a}) = 0 \quad \text{and} \quad \vec{\varepsilon} \cdot \vec{\nabla}\phi_4(\vec{a}) = 0 . \tag{10}$$

Now we can set the spatial coordinate so that the normal vector to the plain, $\vec{\varepsilon}$, has a nonvanishing component only along the x-direction. Then the conditions (10) reduce to

$$\partial_x\phi_3(\vec{a}) = 0 \quad \text{and} \quad \partial_x\phi_4(\vec{a}) = 0 . \tag{11}$$

Assuming that $\partial_x\phi_3$, $\partial_x\phi_4$ and the amplitude of the Higgs field behave independently and denoting the probability of having $\partial_x\phi_j(\vec{a}) = 0$ by d_0, the probability, P_s, that there exist a string-like false vacuum region in the infinitesimal neighborhood at $\vec{x} = \vec{a}$ turns out to be

$$P_s \sim p_0 d_0^2 \ . \tag{12}$$

This is smaller than the case of ordinary topological strings at least by the factor of d_0^2. Obviously we can predict that the more components the Higgs field has, the more difficult it becomes to produce a string, with the higher power of d_0.

For the purpose of estimating p_0 and d_0, we introduce the probability distribution function (PDF) of the Higgs field in the thermal bath. We employ the Hartree approximation [13] with which the higher moment of the field can be described by the second moment. Then the amplitude of a scalar field, ϕ, obeys a random Gaussian probability distribution such as

$$P_\phi \left(\phi\right) d\phi = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{\left(\phi - c\right)^2}{2\sigma^2}\right\} d\phi \ , \tag{13}$$

where c is the the averaged value of ϕ and σ is the standard deviation. Under the same assumption, the gradient of the Higgs field component obeys the PDF

$$P_{\partial\phi} \left(\partial_l \phi_j\right) d\left(\partial_l \phi_j\right) = \frac{1}{\sqrt{2\pi}\eta} \exp\left\{-\frac{\left(\partial_l \phi_j\right)^2}{2\eta^2}\right\} d\left(\partial_l \phi_j\right) \ , \tag{14}$$

where the averaged value of $\partial_l \phi_j$ equals zero. The dispersion, η , which is independent of l, can be written as

$$\eta^2 = \frac{1}{6\pi^2} \int P\left(k\right) k^4 dk \ , \tag{15}$$

together with

$$\sigma^2 = \frac{1}{2\pi^2} \int P\left(k\right) k^2 dk \ , \tag{16}$$

where $P(k)$ is the power spectrum, or the Fourier transform of $\left\langle \phi(\vec{0})\phi(\vec{x}) \right\rangle - c^2$ [14]. Using these formulae, p_0 and d_0 can be written as

$$p_0 = P_\phi\left(0\right) \cdot \delta\phi \ , \quad d_0 = P_{\partial\phi}\left(0\right) \cdot \delta\partial\phi \ , \tag{17}$$

where $\delta\phi$ and $\delta\partial\phi$ are some width scales. The values of σ and η can be obtained by substituting

$$P(k) = \left(k^2 + m_0^2\right)^{-1/2} \left(e^{\frac{\sqrt{k^2+m_0^2}}{T}} - 1\right)^{-1} \tag{18}$$

to the equations (16) and (15) where m_0^2 is the effective mass squared at $\phi = c$ [15].

In the standard electroweak theory, the one-loop effective potential for the Higgs field with the finite temperature corrections is written as [15,16]

$$V_{\rm eff}\left(\phi\right) = D\left(T^2 - T_2^2\right)\phi^2 - ET\phi^3 + \frac{\lambda_T}{4}\phi^4 \ , \tag{19}$$

where T_2 is the temperature when the symmetric state, $\phi = 0$, becomes unstable. Using the standard values of the parameters such as $m_W = 80.6$ GeV for the W-boson mass, $m_Z = 91.2$ GeV for the Z-boson mass, and $m_t = 174$ GeV for the top quark mass, the coefficients in the potential (19) are calculated as $D = 0.169$, $E = 0.00965$, $T_2 = 92.6, 134.3, 249.8$GeV, and $\lambda_{T=T_2} = 0.0354, 0.0747, 0.300$, when the mass of the Higgs particle is $m_H = 60, 100, 200$ GeV, respectively.

We estimate the string formation at the Ginzburg temperature, $T = T_G$, when the defects are considered to turn stable against thermal fluctuations [1,17]. T_G is evaluated by the condition, $T = \Delta V \xi^3$, where ΔV is the potential-energy density gap between the symmetric state and the potential minimum and ξ is the correlation scale of ϕ, which is defined by the square-root inverse of the second derivative of the effective potential at its minimum. Numerically we find

$$T_G = 76.9,\ 62.4,\ 34.4\ \ \text{GeV}\ , \tag{20}$$

for $m_H = 60, 100, 200$ GeV, respectively. Thus T_G is always smaller than T_2, which implies that even if the electroweak phase transition might start as a first-order transition its final stage is described by the dynamics of a second-order phase transition as far as defects formation is concerned.

In the Hartree approximation, the potential (19) is simplified using the replacement

$$\varphi^3 \longrightarrow 3\sigma^2\varphi\ , \quad \varphi^4 \longrightarrow 6\sigma^2\varphi^2 - 3\sigma^4\ , \tag{21}$$

where $\varphi \equiv \phi - c$ and σ is the root mean square of φ which should be equal to the standard deviation in the equation (13). At $T = T_G$, we obtain the effective mass of φ from the coefficient of the quadratic term in the approximate potential as

$$m^2 \equiv m_\varphi^2 + \delta m^2 \ , \tag{22}$$

$$m_\varphi^2 = 2D\left(T_G^2 - T_2^2\right) - 6ET_G c + 3\lambda_{T=T_G} c^2 \quad , \quad \delta m^2 = 3\lambda_{T=T_G}\sigma^2 \ . \tag{23}$$

In order that the expectation value of φ vanishes, or φ has its potential minimum at $\varphi = 0$, the consistency condition for c,

$$2D\left(T_G^2 - T_2^2\right)c - 3ET_G\left(c^2 + \sigma^2\right) + \lambda_{T=T_G} c\left(c^2 + 3\sigma^2\right) = 0 \ , \tag{24}$$

must be satisfied. Now we substitute m into m_0 in (18) and then numerically solve equations (16), (22) and (24) in a self-consistent manner. Using $\lambda_{T=T_G} = 0.0422, 0.103, 0.372$, we find

$$c = 172.4,\ 224.3,\ 237.2 \ \ \text{GeV} \ , \tag{25}$$

$$\sigma = 17.1,\ 9.05,\ 0.717 \ \ \text{GeV} \ , \tag{26}$$

$$m = 46.1,\ 99.6,\ 204 \ \ \text{GeV} \ , \tag{27}$$

and equation (15) yields

$$\eta = 1860,\ 995,\ 67.5 \ \ \text{GeV}^2 \ . \tag{28}$$

Here and hereafter, all the numerical values correspond to the cases $m_H = 60, 100, 200$ GeV, respectively. We can see that $\delta m^2 \ll m_\varphi^2$ justifies the Hartree approximation (21).

p_0 and d_0 are explicitly calculated as

$$p_0 = \left(2.6 \times 10^{-23},\ 1.8 \times 10^{-134},\ 10^{-54682}\right) \times \alpha \ , \tag{29}$$

$$d_0 = (0.168,\ 0.362,\ 0.865) \times \frac{\alpha}{\beta} \ , \tag{30}$$

where we have put $\delta\phi = \alpha\sigma$ and $\delta\partial\phi = \alpha\sigma/\beta m^{-1}$ with α and β being constant. That is, we have normalized $\delta\phi$ by their variance and $\delta\partial\phi$ by $\delta\phi$ divided by the correlation length

$\xi = m^{-1}$. Thus the probability to find a string stretched from $\vec{x} = \vec{a}$ to $\vec{x} = \vec{a} + \vec{\varepsilon}$ is as small as

$$P_s \sim p_0 d_0^2 \sim \left(7.5 \times 10^{-25},\ 2.3 \times 10^{-135},\ 10^{-54682}\right) \times \frac{\alpha^3}{\beta^2}\ . \tag{31}$$

Since $\sigma \ll c$ holds already at the Ginzburg temperature, p_0, which is calculated as (17), turns out to be extremely small. This, however, might not be a fatal problem itself. If false vacuum defects decouple from thermal equilibrium at a higher temperature, say, when σ becomes smaller than c, we should estimate the probability at that temperature. Then $P_\phi(0)$, which is very sensitive to the temperature, could be larger. More serious is the extra suppression factor for a string to extend for a finite length $\epsilon = |\vec{\varepsilon}|$, d_0^2, which is less sensitive to the temperature. For example, for a string to extend for the correlation length

$$\xi = m^{-1} = 0.022,\ 0.010,\ 0.0049\ \mathrm{GeV}^{-1}\ , \tag{32}$$

d_0^2 is as small as

$$d_0^2 = (0.028,\ 0.13,\ 0.75) \times \left(\frac{\alpha}{\beta}\right)^2\ , \tag{33}$$

respectively. But this is not the whole story. Since the discussion based on the lowest-order expansion of $\phi\left(\vec{a} + \vec{\varepsilon}\right)$ is valid only if the inequality

$$\max\left(\delta\phi, \epsilon\delta(\partial\phi)\right) = \max\left(\alpha\sigma, \epsilon\frac{\alpha\sigma}{\beta m^{-1}}\right) > |\varepsilon_i\varepsilon_j\partial_i\partial_j\phi| \sim \epsilon^2\sqrt{\left\langle(\partial^2\phi)^2\right\rangle}\ , \tag{34}$$

is satisfied. Calculating the root-mean square of $\partial^2\phi$ in the same way as in (15), we find

$$\epsilon \lesssim \max\left((0.0059,\ 0.0062,\ 0.0078)\sqrt{\alpha},\ \ (0.0016,\ 0.0039,\ 0.012)\frac{\alpha}{\beta}\right)\ \ \mathrm{GeV}^{-1}\ , \tag{35}$$

which is smaller than or barely comparable to the correlation length of the Higgs field for $\alpha,\ \beta \lesssim \mathcal{O}(1)$. Therefore for a string to extend for the correlation length, we must impose constraints on the amplitude of higher derivatives of ϕ as well, which results in further suppression factor in the formation probability.

One may wonder that in the case of stable nontopological strings there may be a correlation between $\phi = 0$ and $\partial\phi = 0$ and that we may have a larger probability of their formation.

However, previous stability analyses of electroweak strings are all concerned with that of the string width of an infinitely long string solution [4], while strings with finite length are unstable and tend to shrink [6]. Thus the solidness of the string core alone does not help to realize a long string-like configuration. We therefore conclude it is very difficult to find a string longer than the correlation length. In other words, even if a false vacuum string is produced, its length is comparable to its width, and such a configuration should be called a false vacuum ball rather than a string. Thus we cannot make use of such objects for baryogenesis.

In summary, we have considered how difficult it is to produce nontopological defects by the Kibble mechanism in cosmological phase transitions. As a specific example, we have discussed that electroweak strings which are long enough to serve for baryogenesis are very unlikely to be present at the Ginzburg temperature when the defects become stable against thermal fluctuations.

This work was partially supported by the Japanese Grant in Aid for Science Research Fund of the Ministry of Education, Science and Culture (Nos. 5110, 08740202).

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